Rotation problems

Circular motion

Angular and tangential velocity.

1 $\omega = 0.1 \text{ rev s}^{-1} = \frac{\pi}{5} \text{ rad s}^{-1} = 0.628 \text{ rad s}^{-1}$

$v = r\omega = 5.0 \text{ m} \times \frac{\pi}{5} \text{ rad s}^{-1} = \pi \text{ m s}^{-1} = 3.14 \text{ m s}^{-1}$

$\alpha = r\omega^2 = 1.973 \text{ rad s}^{-2}$

Centripetal force and acceleration. Balance of forces.

2 Equating forces $F = \frac{GMm}{r^2} = mr\omega^2$, hence $r^3 = \frac{GM}{\omega^2}$.

For a geostationary orbit there is one revolution in 24 hours, hence $\omega = 7.27 \times 10^{-5} \text{ rad s}^{-1}$, $G = 6.674 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ and $M = 5.98 \times 10^{24} \text{ kg}$ (you can look these up), hence $r = 4.23 \times 10^7 \text{ m}$, about 36000 km above sea level.

3 Equating forces $\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$, cancelling r $\frac{e^2}{4\pi\varepsilon_0 r} = mv^2$, the LHS is minus the potential energy and the RHS is twice the KE, we therefore have 2KE = - PE, and $v^2 = \frac{e^2}{4\pi\varepsilon_0 rm}$.

Rotational kinetic energy.

4 $\alpha = r\omega^2 = 11.8 \text{ m s}^{-2}$, $F = mr\omega^2 = 94.7 \text{ mN}$, $E = \tfrac{1}{2}I\omega^2 = 14.2 \text{ mJ}$

5 $E = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mr^2\omega^2$, $-\left(\frac{\partial E}{\partial r}\right)_\omega = -mr\omega^2$, which is the centripetal force.

Moment of inertia

Moment of inertia (point masses only)

6 $I = \mu r^2 = 5.36 \times 10^{-48} \text{ kg m}^2$, and $\frac{1}{2}I\omega^2 = \frac{2\hbar^2}{I} \Rightarrow \omega = \frac{\hbar}{I} = 3.94 \times 10^{13} \text{ rad s}^{-1}$, time for one cycle is 0.160 ps.

7 (a) The hexagon can be divided into 6 equilateral triangles, thus each C atom is r_{CC} from the centre and each H atom is r_{CC} + r_{CH} from the centre and

$$I = 6\left(m_C r_{CC}^2 + m_H\left(r_{CC} + r_{CH}\right)^2\right) = 2.93\times10^{-45}\ \text{kg m}^2$$

(b) The axis also passes through two H atoms. The remaining 4 C atoms are equidistant from the axis at a distance of $r_{CC}\sin 60° = \dfrac{\sqrt{3}}{2}r_{CC}$, and similarly the H atoms are $\dfrac{\sqrt{3}}{2}\left(r_{CC} + r_{CH}\right)$ from the axis. Thus $I = 4\left(m_C \tfrac{3}{4} r_{CC}^2 + m_H \tfrac{3}{4}\left(r_{CC} + r_{CH}\right)^2\right) = 1.46\times10^{-45}\ \text{kg m}^2$

(c) Four of the C atoms are $\dfrac{1}{2}r_{CC}$ from the axis, and the other two are r_{CC}. Four of the H atoms are $\left(r_{CC} + r_{CH}\right)\sin 30° = \dfrac{1}{2}\left(r_{CC} + r_{CH}\right)$ from the axis and the other two are $\left(r_{CC} + r_{CH}\right)$. Hence

$$I = 4\left(m_C \tfrac{1}{4} r_{CC}^2 + m_H \tfrac{1}{4}\left(r_{CC} + r_{CH}\right)^2\right) + 2\left(m_C r_{CC}^2 + m_H\left(r_{CC} + r_{CH}\right)^2\right) = 1.46\times10^{-45}\ \text{kg m}^2$$

8 (a) The centre of mass is on the stem of the T, 33 pm from the Cl atom and 127 pm from the F atom.

(b) $I = 2.464\times10^{-45}\ \text{kg m}^2$

Angular momentum

Angular momentum.

9 $L = 4.52\times10^{-3}\ \text{kg m}^2\ \text{s}^{-1}$

10 $E = \dfrac{1}{2}I\omega^2 = \dfrac{L^2}{2I}$

Conservation of angular momentum

11 (a) The skater reduces her moment of inertia by pulling her arms in, since angular momentum is approximately conserved and $L = I\omega$, the angular velocity increases.

(b) $I = I_0 + 2mr_1^2$

(c) $L = I\omega = \left(I_0 + 2mr_1^2\right)\omega$ or $\omega = \dfrac{L}{\left(I_0 + 2mr_1^2\right)}$

$$E = \frac{L^2}{2\left(I_0 + 2mr_1^2\right)}$$

(d) Angular momentum *L* does not change, hence $\omega = \dfrac{L}{\left(I_0 + 2mr^2\right)}$ increases as *r* is reduced

and $E = \dfrac{L^2}{2\left(I_0 + 2mr^2\right)}$ increases by the same proportion.

(e) The extra energy must come from work done moving the masses inwards. This is work done in the rotating frame of reference against the centrifugal force.

To show this, $dw = -Fdr = -2mr\omega^2 dr = -\dfrac{2mrL^2}{\left(I_0 + 2mr^2\right)^2} dr$, integrating from r_1 to *r*,

$w = -L^2 \displaystyle\int_{r_1}^{r} \frac{2mr}{\left(I_0 + 2mr^2\right)^2} dr = L^2 \left[\frac{1}{2\left(I_0 + 2mr^2\right)}\right]_{r_1}^{r} = \frac{L^2}{2\left(I_0 + 2mr^2\right)} - \frac{L^2}{2\left(I_0 + 2mr_1^2\right)}$, which is the same as the difference in kinetic energy $E(r) - E(r_1)$.

12 Because of air resistance the main rotor operating at a constant angular velocity exerts a torque (twisting force) on the air. By Newton's third law, the air exerts an equal and opposite torque on the helicopter, so that without something to balance this the helicopter would start to rotate in the opposite direction to the main rotor. The tail rotor exerts a force away from the centre of lift, i.e. a torque, to balance this effect enabling the helicopter to point in a constant controllable direction. The torque of the tail rotor also allows the pilot to re-orient the helicopter.

13 Either $4\hbar$ or $2\hbar$.